Steady state heat transport in magnetized plasmas with magnetic islands and local stochastic fields

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Motivation

Model

Magnetic islands

Stochastic layers

Summary
Motivation
Motivation: Magnetic islands

- magnetic reconnection (example: 3/2-island)
- field lines wander around island surfaces
- parallel transport (χ∥): fast, long distance
- perpendicular transport (χ⊥): slow, short distance
- parallel + perp. transport \( \Rightarrow \) temperature flattening
- scale island width \( w_c \): parallel \( \approx \) perpendicular transport
Motivation: Neoclassical tearing mode

- bootstrap current: toroidal current driven by $\nabla_r p$
- perturbed by island temperature flattening
  - effective lack current in the island o-point region
  - acts as a driving term for further island growth
  - $\Rightarrow$ neoclassical tearing mode (NTM)
- NTM stability strongly depends on temp. distribution
- exact heat flux computations are important
- analytical theory is limited to the cases $w/w_c \to 0$ and $w/w_c \to \infty$, see Fitzpatrick (1995)
- AIM: numerical computations for realistic parameters
Motivation: Stochastic layers

- overlapping magnetic islands destroy flux surfaces

- field lines move through stochastic layer in a seemingly random way (increases radial heat transport)

- AIM: determine radial heat conductivity of highly stochastic layers and compare to analytical predictions
Model
Model: Geometry

- equilibrium: circular cross section, large aspect ratio
- $q$-profile with 0.9 in the center and 4.0 at the edge

- coordinates: helical, unsheared, rational helicity $q_c$
  (adapted to problem)
- grid points in radial and poloidal direction
- Fourier expansion in toroidal direction
Model: Heat diffusion equation

- Solve steady state heat diffusion equation $\nabla \vec{q} = P$
  - $P$: power source (heating)
  - $\vec{q} = -n\chi_\parallel \nabla_\parallel T - n\chi_\perp \nabla_\perp T$: heat flux density
  - $n$: particle density
  - $\chi_\parallel / \chi_\perp$ typically between $10^7$ and $10^{11}$
- common numerical schemes: error $\propto \chi_\parallel / \chi_\perp \Rightarrow$ exact coordinate alignment
- virtually impossible for time-dependent problems
- new scheme developed by Günter et al. (2005)
  - conserves self-adjointness of parallel transport operator
  - temperature and heat flux grids shifted against each other
  - Fourier cut-off is performed at a certain heat flux order
Model: New numerical scheme

- deviations from analytical solution for non-trivial test case
- small numerical errors
- independent from $\chi_\parallel/\chi_\perp$ up to $10^{13}$

- code benchmarks were also performed
Magnetic island results
Magnetic islands: Radial heat diffusivity $\chi_r$

- effective radial heat diffusivity $\chi_r$
  - increased in the island region
  - maximum at the resonant surface
- 4/3-island with $w = 0.068a$: 

![Graph showing radial heat diffusivity $\chi_r$ as a function of $r/a$.]
Magnetic islands: Scaling of $\chi_r$

- $\chi_r$ at the island resonant surface

- Two different regimes: $\chi_r \propto \left(\frac{w}{w_c}\right)^4$ resp. $\chi_r \propto \left(\frac{w}{w_c}\right)^2$

- Depends on $w/w_c$ only
Magnetic islands: Heat flux

- Total heat flux in the island region ($4/3$-island, $w = 0.068a$)
- $w/w_c = 3.4 \Rightarrow$ largely flattened island
Magnetic islands: Heat flux components

- Same case
- Parallel (red) and perpendicular (blue) heat flux
NTM stability: Comparison to analytical limits

- NTMs destabilized by seed island temperature pert.
- Neoclassical contribution to island growth rate:

Numerical results match analytical limits
NTM stability: Comparison to analytical matching

- Fitzpatrick performed matching of the analytical limits

- underestimates island growth rate significantly
- can make the difference between stable and unstable
Results for highly stochastic layers
Stochastic layers: Flux surface destruction

- flux surfaces destroyed for
  \[ s = \frac{(w_1 + w_2)/2}{|r_{res,1} - r_{res,2}|} \gtrsim 1 \]
- test case:
  - 5 islands: 24/23, 25/24, 26/25, 27/26, 28/27
  - total stochasticity \( s = 48.5 \gg 1 \)
Stochastic layer: Radial heat diffusivity $\chi_r$

- radial heat diffusivity $\chi_r$ strongly increased in the region of the stochastic layer
Stochastic layers: Analytical theories

- analytical theories for heat transport across highly stochastic layers ($s \gg 1$): review by Liewer (1985)
- Rechester-Rosenbluth regime
  - low collisionality
  - electrons basically follow the stochastic field lines
  - transport dominated by field line diffusion
- Kadomtsev-Pogutse regime
  - medium collisionality
  - increased importance of electron diffusion
- fluid regime
  - high collisionality
  - transport dominated by electron diffusion
Stochastic layer: Scaling of $\chi_r$

- $\chi_r$ in the center of the ergodic layer:

- the three analytically predicted regimes can be observed
- ranges of validity do not coincide
Summary

- implemented code for heat diffusion computations
- demonstrated computations with unaligned coordinates
- radial heat diffusivity $\chi_r$ for islands
- $[dw/dt]_{bs}$ for neoclassical tearing modes; widely used analytical matching underestimates island growth!
- $\chi_r$ at highly stochastic layers with realistic parameters
  - found the analytically predicted regimes
  - different ranges of validity
- experience from this work will be used to implement a nonlinear MHD code for plasma edge examinations
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References
Island code benchmark with TM1

- 1/1 magnetic perturbation with $w = 0.118a$
- at $\chi_\parallel/\chi_\perp = 10^{11}$: $w/w_c = 13.8$
Ergodic code benchmark with TM1

- $4/3$ and $3/2$ magnetic perturbations
- $s = 1.4$
- $w/w_c$ between 0.34 and 3.4
- Relative differences for representative modes:
Analytical test case

\[ \vec{B} = \nabla \psi \times \hat{e}_\phi, \]  
1

\[ \psi(r, \theta, \phi) = \psi_0(r) + \psi_1(r, \theta, \phi), \]  
2

\[ \psi_0(r) = \psi_0 \left( \frac{r}{a} \right)^2 \left( \frac{r - r_0}{a} \right)^2, \]  
3

\[ \psi_1(r, \theta, \phi) = \psi_1 \left( \frac{r}{a} \right)^2 \cos(m\theta - n\phi). \]  
4

- analytical solution:

\[ T(r, \theta, \phi) = \psi(r, \theta, \phi), \]  
5
Analytical test case (continued)

- boundary condition: \( T(r = a) = \Psi(r = a) \)
- analytical solution: parallel heat flux \( \vec{q}_\parallel = \hat{b}(\hat{b} \cdot \nabla T) \)
  vanishes as \( (\nabla \Psi \times \hat{e}_\phi) \cdot (\nabla \Psi) = 0 \)
- reduces heat flux to \( \vec{q} = \vec{q}_\perp = -n_{\chi\perp} \nabla T \)
- heat diffusion equation \( \nabla \cdot \vec{q} = -P \Rightarrow P = n_{\chi\perp} \nabla \cdot \nabla \Psi \) (to get the analytic solution)
- code runs with \( \psi_1/\psi_0 = 10^{-2}, \ m = 3, \ n = 2, \ r_0/a = 1.2 \)
- resolution: 160 \times 160 grid points
- error: \( \text{Err}_{0,0} = \left[ \frac{T_{\text{num}} - T_{\text{analyt}}}{T_{\text{analyt}}} \right]^{0,0} \)
  \( r = 0.5a \)
Scale island width $w_c$

- competition between parallel and perp. transport
- for $w \approx w_c$, parallel $\approx$ perpendicular transport
- scale island width $w_c = r_{\text{res}} \left( \frac{\chi_{\|}}{\chi_{\perp}} \right)^{-1/4} \sqrt{\frac{8}{\epsilon_s s_s n}}$
  - $n$: toroidal mode number of perturbation
  - $s_s$: local magnetic shear
  - $r_{\text{res}}$: minor radius of resonant surface
  - $\epsilon_s = r_{\text{res}}/R$: local inverse aspect ratio (R: major radius)
Finite difference scheme

- Temperature grid (blue) and heat flux grid (green).

- Differential operators discretized with symm. 2\textsuperscript{nd} order form
- e.g., the radial gradient of the temperature:

\[
\begin{bmatrix}
\frac{\partial T}{\partial r}
\end{bmatrix}_{i+\frac{1}{2},j+\frac{1}{2}} = \frac{T_{i+1,j+1} + T_{i+1,j} - T_{i,j+1} - T_{i,j}}{2\Delta r} + \mathcal{O}(\Delta r^2), \quad (6)
\]