Heat Diffusion
in realistic Tokamak Geometry

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Outline

1 Motivation

2 Model
   - Heat Diffusion Equation
   - Coordinate system
   - Coordinate alignment

3 Magnetic Islands
   - Temperature flattening
   - Comparison to TEXTOR

4 Ergodic Layers
   - Temperature flattening
   - FIR-NTMs

5 Edge Ergodization

6 Summary
Motivation

Safety Factor

“Safety factor” \( q \): Number of toroidal turns per poloidal turn

\[ \nu = 1/q \]
Poincaré plot: Field lines traced for many toroidal turns
2/1 magnetic island at $q = 2$ surface
Heat Diffusion in realistic Tokamak Geometry

Motivation

Temperature flattening

- Temperature profile flattens inside the magnetic island
- Bootstrap current $\propto \nabla p$ perturbed $\Rightarrow$ Island drive (NTM)

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Model

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Steady State Heat Diffusion Equation

$$\nabla \cdot \mathbf{q} = P,$$ where $$\mathbf{q} = -n_e \left( \chi_\parallel \nabla_\parallel T + \chi_\perp \nabla_\perp T \right)$$

$q$: heat flux, $P$: energy source, $n_e$: electron density, $\chi_\parallel$ and $\chi_\perp$: heat diffusivities

Anisotropy

$$\chi \equiv \chi_\parallel / \chi_\perp \approx 10^8 \ldots 10^{10}$$

Finite Difference Scheme

see Günter et al. (2005)

- Two staggered grids
- Low numerical diffusion
- Coordinate alignment not required
- Realistic anisotropies
Curvilinear Coordinate System

- Heat diffusion eq. in tensor notation:

\[
\frac{1}{\sqrt{g}} \frac{\partial}{\partial u^\alpha} (\sqrt{g} q^\alpha) = P
\]

\[
q^\alpha = -n_e \chi_\perp \left[ \chi b^\alpha b^\beta + g^{\alpha\beta} \right] \frac{\partial T}{\partial u^\beta}
\]

- \(q^\alpha\): Contravariant heat flux components
- \(u^\alpha\): Contravariant coordinates (\(\rho\), \(\theta\), \(\phi\))
- \(g^{\alpha\beta}\): Metric tensor components
- \(g = \det[g^{\alpha\beta}]\): Determinant of the covariant metric tensor

- Axisymmetric straight field line coordinates
Coordinate Alignment to Unperturbed Magnetic Field

- Coordinate Transformation \( \theta = \tilde{\theta} - \iota \cdot \phi \)
  \( \Rightarrow \) Sheared helical coordinates

\( \iota = 1/q \): Inverse safety factor

- Problems:
  - Grid deformation
  - Interpolation for toroidal periodicity condition \( T_{\phi=0} \equiv T_{\phi=2\pi} \)
    increases numerical diffusion
  - Restriction \( \chi_{||}/\chi_{\perp} \lesssim 10^7 \) \( \times \)
Partial Coordinate Alignment

- Transformation $\theta = \tilde{\theta} - \iota_c \cdot \phi$
  \[ \iota_c \equiv \text{const} \]
  $\Rightarrow$ Unsheared helical coordinates

- Realistic anisotropies ✔

Restrictions due to the Misalignment?

- Islands resolved well for: $N_\phi \gtrsim \Delta t \cdot N_\theta$
  \[ \Delta t = |t - \iota_c|: \text{misalignment at island} \]
  $N_\phi, N_\theta$: toroidal and poloidal grid point numbers

- Suitable for magnetic perturbations with similar helicities
- Islands with very different helicities increase the numerical effort
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Magnetic Islands

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Heat Transport across Magnetic Islands

see Fitzpatrick (1995); Yu (2006); Hölz et al. (2007)

- Scale island width \( w_c \propto (\chi_\parallel / \chi_\perp)^{-0.25} \)

\[
\begin{align*}
\frac{w}{w_c} \begin{cases} 
\ll 1 & \text{No perturbation} \\
\gg 1 & \text{Temperature flattening}
\end{cases}
\end{align*}
\]

- Heat conduction layer at the separatrix
- Temperature flattening destabilizes NTMs (perturbation of the bootstrap current)
- This talk: Realistic tokamak geometry
Heat Diffusion in realistic Tokamak Geometry

Magnetic Islands

Temperature flattening

- ASDEX Upgrade equilibrium
- 3/2 island with $w = 8.1$cm

$\theta = 0$: Low field side  $\theta = \pi$: High field side

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Magnetic Islands

Temperature flattening

- ASDEX Upgrade equilibrium
- 3/2 island with $w = 8.1\text{cm}$

$\theta = 0$: Low field side  $\theta = \pi$: High field side
Comparison to TEXTOR (preliminary)

Data provided by Ivo Classen (see Classen (2007))

- 2/1 island triggered by DED coils
- Mode frequency 1 kHz
- ECE frequency 100 kHz
- Channels cover part of the island (including x-point)
- Channels not cross-calibrated
- Comparing during growth phase

Aim: Draw conclusions for experimental $\chi_{||}/\chi_{\perp}$

TEXTOR: Tokamak experiment in Jülich with a circular plasma cross section

DED coils: Set of perturbation coils at TEXTOR (Dynamic ergodic divertor)

ECE: Diagnostic measuring the electron temperature (Electron cyclotron emission spectroscopy)
Timetrace of an ECE channel

TEXTOR #99175 (channel 05-08)
Numerical simulation

- Code runs with different $\chi_{||}/\chi_{\perp}$, power source, energy source, ...
- Toroidal temperature cuts:
Automatic matching

- Adding calibration-summands to the ECE channel signals
- Best-fitting numerical code run for each experimental timepoint

![Simulation plot](image-url)
Problems

- Sudden change of the island structure as the mode locks to the DED perturbation field
- Best fitting $\chi_\parallel / \chi_\perp$ changes strongly

Reasons?

- Perturbation profile?
- Higher harmonics (4/2, . . .)?
- Different modes excited by DED coils (3/1, . . .)?

Additional Comparisons planned

- ECRH heating at magnetic island
- ASDEX Upgrade with new ECE diagnostic
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Ergodic Layers

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Ringberg Theory
Heat Diffusion across an Ergodic Layer

- Overlapping magnetic islands produce an ergodic layer
- Chaotic field line trajectories

\[ \frac{\chi_{||}}{\chi_{\perp}} = \begin{cases} 
\text{small: single island effects dominate} \\
\text{large: ergodisation increases transport} 
\end{cases} \]
- Overlapping $3/2$ and $4/3$ islands
- Chirikov parameter $\sigma_{Ch} = 1.52$

$\theta = 0$: Low field side  \quad \theta = \pi$: High field side
Overlapping 3/2 and 4/3 islands

Chirikov parameter $\sigma_{Ch} = 1.52$

$\theta = 0$: Low field side \hspace{1cm} \theta = \pi$: High field side
FIR-NTMs

FIR-NTM: Neoclassical tearing mode in the frequently interrupted regime

see Günter et al. (2001) and Gude et al. (2002)

- Frequent interruption of NTM growth by sudden amplitude drop
- Much faster than the resistive timescale
- Observed at large normalized plasma pressure $\beta_N$ (i.e. large bootstrap current fraction)

![Graph showing even n signal over time](ASDEX Upgrade #10192)
**FIR-NTMs**

- Island $\Rightarrow$ T flattening $\Rightarrow$ bootstrap current perturbation $\Rightarrow$ NTM
- Considering $3/2$ NTM and additional $4/3$ perturbation
- Resonant bootstrap current perturbation strongly reduced for $\chi_{||}/\chi_{\perp} \gtrsim 1 \cdot 10^9$ and $\sigma_{Ch} \gtrsim 1.4 \Rightarrow$ Less island drive
- $4/3$ perturbation expected to be ideal (timescale!)

\[ \begin{array}{c}
\chi_{||}/\chi_{\perp} = 4 \cdot 10^6 \\
\chi_{||}/\chi_{\perp} = 7 \cdot 10^7 \\
\chi_{||}/\chi_{\perp} = 1 \cdot 10^9 
\end{array} \]
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Edge ergodization

- Perturbation coils planned for ASDEX Upgrade
- Among others aimed at the mitigation of edge localized modes (ELMs)
- Ergodization of the plasma edge
- Increased heat conduction due to the transport of electrons along magnetic field lines
Spitzer conductivity assumed
Significant drop of edge temperature gradient observed
Very sensitive to plasma parameters!
Summary

- Unsheared helical coordinates
- Realistic $\chi_\parallel/\chi_\perp$ possible (islands, ergodic layers, ergodic edge)
- Magnetic islands: Temperature flattening for $w/w_c \gtrsim 2$
- Comparison to TEXTOR
  - ECE timetraces vs. toroidal cuts
  - Automatic matching
  - Problems with perturbation profile
- Ergodic layers
  - Temperature flattening at the ergodic layer for large $\chi_\parallel/\chi_\perp$
- NTM
  - Resonant bootstrap current perturbation drives island
  - FIR-NTM: Frequent amplitude drop
  - Possible explanation: Ergodization reduces island drive
- Edge: Ergodization might increase radial heat transport
Thanks for your attention!

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References

Most of the results shown in this talk have been published in Hölzl et al. (2008).


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