Non-linear ELM and RMP Modeling in Realistic Tokamak Geometries

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What is an Edge Localised Mode?

Periodic relaxation of heat & density

\[ \rightarrow \text{steep } \nabla p \text{ (ballooning)} \]

\[ \rightarrow \text{large edge current (peeling)} \]

MAST fast CCD camera [Kirk '06]
ELMs are harmful instabilities for the PFCs —the need to understand, predict & control ELMs in realistic geom.—

ELM power load: $e^-$ gun @Kurchatov

Limits pedestal height & global confinement

- erosion, droplets, melting of tungsten
- Q=10 in ITER: $\Delta W_{ELM}^{ITER} \sim 17 \text{MJ} \sim 15\% \ W_{ped}$
  - in $\sim 250-500 \ \mu s$
- acceptable ELM: $\Delta W_{ELM} \sim 2 - 3 \text{MJ}$
  - $\downarrow$ divertor may only survive a few ELMs...

- $e^-$ gun power load based on empirical extrapolation —not understood
- power load cycles (~1000) at low power show intense material degradation
ELMs are harmful instabilities for the PFCs
— the need to understand, predict & control ELMs in realistic geom.—

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- acceptable ELM: \( \Delta W_{ELM} \sim 2 - 3 \text{MJ} \)
  \[ \Rightarrow \] divertor may only survive a few ELMs...

**Common thread:** what requirements for an accurate description of ELMs & RMPs?

1. the tool: the reduced MHD code \textsc{JOREK}
2. evaluating the ELM energy deposition in \textsc{ITER}
3. added physics: diamagnetics & RMPs
4. going further: divertor physics

ELMy power load: \( e^- \) gun @Kurchatov

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*Guilhem Dif-Pradalier*

ICNSP ★ Beijing ★ September 2013
Requirements for an accurate description of ELMs & RMPs

1. the tool: the reduced MHD code JOREK
2. The ELM energy deposition in ITER
3. Added physics: diamagnetics & RMPs
4. Going further: divertor physics
JOREK: developed with the specific aim to simulate ELMs

- Originally developed at CEA Cadarache \[\text{[Huijsmans '07, Czarny '08]}\]
- Non-linear reduced MHD in toroidal geometry
  \[\text{next slide: dens., temp., elec. potential (perp. flow), para. velocity, polo. flux]}\]
- Full MHD in development

- Domain:
  - Closed & open field lines, X-point
  - b.c.: Mach one, free outflow at divertor target

- Discretisation:
  - Cubic Bezier finite elements in the poloidal plane
  - Fourier series in toroidal angle

- Time stepping: fully implicit Crank-Nicholson

- Solver sparse matrices (PastiX library): \(10^7\) degrees of freedom

- Parallelisation using MPI/OpenMP: typically 256 — 1500 processors

- Pellet ELM triggering \[\text{[Huijsmans '10]}\]
- ELMs in JET \[\text{[Pamela '11]}\]
- RMP field penetration \[\text{[Bécoulet '12, Orain '13]}\]
R-MHD equations, including: SOL flows, source $S_{\nu \varphi}$, two-fluid **diamagnetic** rotation & **NC poloidal** viscosity

1—density: 
\[
\frac{\partial}{\partial t} \rho = - \nabla \cdot (\rho \mathbf{v}) + \nabla \cdot (D_\perp \nabla_\perp \rho) + S_\rho \]

[Huijsmans ’09, Orain ’13]

2—temperature: 
\[
\rho \frac{\partial}{\partial t} T = -\rho \mathbf{v} \cdot \nabla T - (\gamma - 1) \rho T \nabla \cdot \mathbf{v} + \nabla \cdot (\kappa_\perp \nabla_\perp T + \kappa_\parallel \nabla_\parallel T) + S_T
\]

3—perp. and parallel momentum: 
\[
\mathbf{e}_\varphi \cdot \nabla \times \left( \rho \frac{\partial}{\partial t} \mathbf{v} = -\rho (\mathbf{v} \cdot \nabla) \mathbf{v} - \nabla (\rho T) + \mathbf{J} \times \mathbf{B} + \mu \Delta \mathbf{v} \quad -\nabla \cdot \Pi^{\text{neo}} + S_{\nu \varphi} \right)
\]

4—induction: 
\[
\frac{\partial}{\partial t} \mathbf{A} = -\eta \mathbf{J} \quad -\frac{m}{\rho e} \nabla_\parallel (\rho T) + \mathbf{v} \times \mathbf{B} - F_0 \nabla \phi
\]

5—B field & closure: 
\[
\mathbf{B} = \frac{F_0}{R} \mathbf{e}_\varphi + \frac{\nabla \psi(t)}{R} \times \mathbf{e}_\varphi \quad ; \quad \eta = \eta_0 \left( \frac{T}{T_0} \right)^{-3/2} \quad ; \quad \mathbf{v} = -R \nabla \phi(t) \times \mathbf{e}_\varphi + v_\parallel(t) \mathbf{B} + \mathbf{v}_* 
\]

7—boundary conditions:

- Zero perturbations on wall aligned with last flux surface
- Bohm boundary conditions on the target: $v_\parallel = \pm c_s$ ; $\kappa_\parallel \mathbf{b} \cdot \nabla T = (\gamma - 1) n T c_s$
A typical run — e.g. in ITER geometry

- Initial grid: polar grid for Bézier elements
- Flux-aligned grid including X-point(s)
- Radial and poloidal grid meshing: divertor & wall b.c.
- Equilibrium flows: $n = 0$ harmonic
- Time-integration: $\forall n$ harmonics
- Postprocessing
A typical run —e.g. in ITER geometry

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Requirements for an accurate description of ELMs & RMPs

1. the tool: the reduced MHD code JOREK
2. The ELM energy deposition in ITER
3. Added physics: diamagnetics & RMPs
4. Going further: divertor physics
ELM size, wetted area & peak heat/particle load: the prediction for ITER is both uncertain & crucial

F4E-GRT265:

« Evaluation of edge MHD stability and uncontrolled ELM energy losses for ITER H-mode plasmas in non-active, DD and DT operational scenarios »

Known limitations: a realistic ELM computation in ITER is yet out-of-scope.

State-of-the-art: preliminary attempt to compute heat & particle deposition in 15MA/5.3T ITER

[Maget '12, Dif-Pradalier & Bécoulet '13]
State-of-the-art...yet many unknowns

- **particle loss in ELM:** \( \sim 3.4\% \)
- **energy loss in ELM:** 5MJ out of 452.5MJ \( \sim 1.1\% \) energy content

Going beyond...

- **grid:**
  - aligned v.s. adaptive [Ratnani]
  - low \( \eta_0 \Rightarrow \) large grid \( \Rightarrow \) memory
- **memory:** multi-harmonics needed for turb. & \( \mathbf{E} \times \mathbf{B} \) shear \( \Rightarrow \) large in implicit models
- **time-stepping:** fast parallel dyn. v.s. perp.
- **boundaries:** interaction with chamber magn. connection, free boundary [STARWALL]

\[ \begin{array}{|c|c|c|}
\hline
\text{Resistivity} & \eta_0 = 10^{-6} & 10^{-10} \\
\text{Parallel/perp. heat cond.} & \kappa || / \kappa _\perp = 810^8 & 10^{11} \\
\hline
\end{array} \]

\( \Rightarrow \) how does the ELM computation change when adding new physics?
Requirements for an accurate description of ELMs & RMPs

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Diamagnetic rotation $\omega_\star$ seems instrumental to access crash cycles.

**why a cycle?**

- **usually:** initial unstable profiles $\nabla p$, $I$ $\rightarrow$ single relaxation
- **1st relax.:** "unphysical"? $\rightarrow$ analog. sawteeth [q–profile, reconn. dyn.] [Lütjens '09]
- **assess dynamics with** self-consistent background flows, electric field & mode phasing

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**Energy of the ELM (n=6 only)**

- $\omega_\star = 0$

**Kinetic energy of the modes n=2,4,6,8**

- $\omega_\star \neq 0$

[Orain '13]
Ongoing work: diamagnetic rotation reduces the ELM size & frequency

- when including dia. rotation...
  - heat deposited on divertor is reduced when $\omega_* \neq 0$

Power on divertor

<table>
<thead>
<tr>
<th>$t/t_A$</th>
<th>Integrated power (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2500</td>
<td>$10^0$</td>
</tr>
<tr>
<td>3000</td>
<td>$10^1$</td>
</tr>
<tr>
<td>3500</td>
<td>$10^2$</td>
</tr>
<tr>
<td>4000</td>
<td>$10^3$</td>
</tr>
<tr>
<td>4500</td>
<td>$10^2$</td>
</tr>
<tr>
<td>5000</td>
<td>$10^1$</td>
</tr>
</tbody>
</table>

$\omega^* = 0$

$\omega^* = \omega^*_\text{ref}$

ELM freq. ↑ when $\omega_* \downarrow$

- ELM size & dynamics  → 2-fluid dia. rotation important

$W_{\text{mag}}$ of the mode $n=6$: $\omega^*$ scan

$\omega^*_\text{ref}/2$

$\omega^*_\text{ref}$

[Orain '13]

Guilhem DIF-PRADALIER
RMPs: some very contrasted results...beg for a better understanding

- External coils apply static Resonant Mag. Perturb.
- ergodic edge
- radial transport
- $\nabla p < $ instability threshold

- same “vacuum ergodization”, different effects: suppress, mitigate, trigger, ...

- RMPs with plasma response
- RMPs / ELMs interaction
- Density pump-out
- Rotation braking

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Ongoing work: ELMs are mitigated $\Rightarrow$ power in divertor is $\sim 10x$ smaller with RMPs

Power [in MW] deposited on the divertor

[Bécoulet '13]

[Diagram showing power deposition over time with and without RMPs]

with RMPs

outer divertor

inner divertor

without RMPs
Requirements for an accurate description of ELMs & RMPs

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Importing divertor physics: sources, neutrals & geometry ➔ a strong impact on the SOL flows

- Flows ➔ strong influence on onset & development of instabilities  
  ➔ supersonic transitions in JOREK: a surprise?
- \{S, neutrals, geom.\} ➔ strong influence on onset & structure of flows

Supersonic transition in the SOL driven by plasma source inversion
Importing divertor physics: sources, neutrals & geometry → a strong impact on the SOL flows

- Flows → strong influence on onset & development of instabilities
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Supersonic transition in the SOL driven by plasma source inversion

\[ \mathcal{A} = \frac{2M}{1 + M^2} \quad \text{[Ghendrih '12, Bufferand '13]} \]

\[ \frac{\partial \mathcal{A}}{\partial s} \propto (S_{\text{perp}} + S_{\text{ionization}}) \]

- If \( \partial_s \mathcal{A} \leq 0 \) target plates \( \Rightarrow M \geq 1 \) satisfy Bohm

- \( S_{\text{perp}} + S_{\text{ioniz.}} \) near target plates
  - \( < 0 \) → supersonic
  - \( > 0 \) → subsonic

\[ S_{\text{perp}} > 0 \]

**supersonic flows**
Importing divertor physics: sources, neutrals & geometry ➔ strong impact on the SOL flows

- Flows ➔ strong influence on onset & development of instabilities
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- \( S_{\text{perp}} + S_{\text{ioniz.}} \)
  - near target plates
  - \( < 0 \) ➔ supersonic
  - \( > 0 \) ➔ subsonic

NO supersonic flows
Importing divertor physics: sources, neutrals & geometry → a strong impact on the SOL flows

- Flows → strong influence on onset & development of instabilities [see $\omega_*$]
  ↓ supersonic transitions in JOREK: a surprise?
- \{S, neutrals, geom.\} → strong influence on onset & structure of flows

Supersonic transition in the SOL driven by plasma source inversion

$$A = \frac{2M}{1+M^2} \quad [\text{Ghendrih '12, Bufferand '13}]$$

$$\frac{\partial A}{\partial s} \propto (S_{\text{perp}} + S_{\text{ionization}})$$

- If $\partial_s A \leq 0$ near target plates $\Rightarrow M \geq 1$ satisfy Bohm

- $S_{\text{perp}} + S_{\text{ioniz.}} < 0$ $\Rightarrow$ supersonic
  $\Rightarrow$ supersonic flows

- $S_{\text{perp}} + S_{\text{ioniz.}} > 0$ $\Rightarrow$ subsonic

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SolEdge2D $\{n, u_\parallel, T\} +$ neutrals $\rightarrow$ supersonic transitions understood in the SOL

[Bufferand '13]

Ionisation

low density, ITER

high density, ITER

detached, WEST

Mach$_\parallel$

supersonic subsonic supersonic

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Conclusions

First series of ELM computation for iter $[0^{th} \text{ order}]$, validated by ITER

Diamagnetic flows $\omega_\star$:
- seem essential for ELM cycles
- reduce ELM size, increase freq.
- symmetrisation of the power deposition

ELM mitigation by RMPs

Supersonic flow transitions in SOL — divertor physics
- framework understanding becoming mature
- delicate balance: sources, neutrals [ionis.], $B$ geom. ➔ all effects important

many numerical challenges remain, e.g. talk tomorrow by A. Ratnani
Additional material
JOREK in a nutshell

- non-linear reduced MHD in toroidal geometry
  - density, temperature, velocity & poloidal flux
  - ideal wall conditions on walls
  - Mach one, free outflow at divertor target

- closed & open field lines domain, X-point geom.
  - cubic finite elements, flux aligned poloidal grid
  - Fourier series in toroidal direction

- time stepping, solver & parallelism
  - fully implicit Crank-Nicholson
  - sparse matrices (PastiX): $\sim 10^7$ degrees of freedom
  - MPI/OpenMP over typically 256 – 1500 processors

- getting closer to the experiment...
  - exact geometry** & boundary conditions**
  - non-linear MHD over long time scales* ($\mu s \rightarrow s$)
  - realistic parameters*** [resistivity, parallel conductivity, collisionality]
  - one/several modes***, background turbulence****
JOREK, a European network to study edge MHD instabilities: ELMs & disruptions

- **ELM cycle & control**
  - ELMs  
    [G. Dif-Pradalier, M. Bécoulet, S. Pamela]
  - Resonant Magnetic Perturbations (RMPs)  
    [M. Bécoulet, F. Orain]
  - pellets injection, vertical kicks  
    [G. Huijsmans, S. Futatani]

- **Disruptions**
  - VDE, $\beta$ limit disruptions, density limit  
    [C. Reux, E. Nardon, A. Fil]
  - NTMs control with ECCD  
    [IO+FOM]

**ANRs:** ASTER (2006-2009), ANIKA (2009-2012), ANEMOS (2010-2013), A2T2 (2010-2013)

**Grants:** F4E-2011-GRT-265

“Jorek team”: ~30 throughout Europe

International:
- **JOREK**  
  [Huijsmans ‘07]
- **M3D–C1**  
  [Ferraro ‘09]
- **XGC0**  
  [Park ‘07]
- **BOUT++**  
  [Dudson ‘09]
ELM size, wetted area & peak heat/particle load: the prediction for ITER is both uncertain & crucial

**ELM ≡ MHD instability destabilised by pressure & current gradients in the H-mode pedestal**

- **stringent operational limits:** $W_{ELM}/W_{ped} \sim 15\%$ in $\sim 250$-$500$ µs

- **ELM energy content:** $W_{ELM}$ ↗ when coll. ↓ [Loarte ’03, Pamela ’10, Zarzoso ’11]

- **ELM energy deposition area:** does the power density ↗ when $W_{ELM}$ ↗?

- **peak heat load localisation:** changes during ELM [Thomsen ’10]

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**Gr#265 ≡ study these aspects in realistic iter geometry and standard [15MA, 6keV] scenario**

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Physics forewords. An acceptable ELM in ITER?

[ELM power loads: conservative; broadening not taken into account]

- Uncontrolled ELMs in ITER: $\sim 20\text{MJ at } 15\text{MA}, Q = 10$
  - acceptable limit for material damage: $0.5\ \text{MJ m}^{-2}, \Delta W_{\text{ELM}}^\text{contr.} \sim 0.7\ \text{MJ}$

- A significant broadening of ELM footprint could increase uncontrolled ELM operation from $6\text{MA} (A_{\text{ELM}} = A_{ss})$ to $9\text{MA} (A_{\text{ELM}} = 4A_{ss})$
  - No large influence on ELM size limit at $15\text{MA}$ (small ELMs)

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Ongoing effort: assess energy & particle deposition for Iter

- realistic parameters challenging: $\nu_*$, resistivity $\eta$, transp. anisotropy $\chi_{||}/\chi_{\perp}$, size, shape...
*State-of-the-art...yet many unknowns*

- **particle loss in ELM:** $\sim 3.4\%$
- **energy loss in ELM:** 5MJ out of 452.5MJ $\sim 1.1\%$ energy content
- **near-symmetric power deposition for a large ELM**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistivity $\eta_0$</td>
<td>$10^{-6}$</td>
<td>$10^{-10}$</td>
</tr>
<tr>
<td>Parallel/perp. heat cond. $\kappa_{\parallel}/\kappa_{\perp}$</td>
<td>$810^8$</td>
<td>$10^{11}$</td>
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<tr>
<td>Diamagnetics</td>
<td>none</td>
<td></td>
</tr>
<tr>
<td>Neoclassics</td>
<td>none</td>
<td></td>
</tr>
<tr>
<td>Neutrons</td>
<td>none</td>
<td></td>
</tr>
<tr>
<td>Radiation</td>
<td>none</td>
<td></td>
</tr>
<tr>
<td>Harmonics</td>
<td>single $[n = 9]$</td>
<td></td>
</tr>
<tr>
<td>ELM cycle</td>
<td>single relax.</td>
<td></td>
</tr>
</tbody>
</table>

What happens when relaxing some of the above limitations?