Nonlinear Reduced Magnetohydrodynamic Simulations of Edge-Localized Modes in Tokamak Plasmas

Isabel Krebs

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Introduction – JOREK & ELMs

ELM simulations

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Introduction

JOREK: reduced MHD

- JOREK solves nonlinear reduced MHD equations in toroidal geometry
JORÉK solves nonlinear reduced MHD equations in toroidal geometry

\[
\frac{\partial \Psi}{\partial t} = \eta j - R [u, \Psi] - F_0 \frac{\partial u}{\partial \phi}
\]

\[
\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho v) + \nabla \cdot (D_\perp \nabla_\perp \rho) + S_\rho
\]

\[
\frac{\partial (\rho T)}{\partial t} = -v \cdot \nabla (\rho T) - \gamma \rho T \nabla \cdot v + \nabla \cdot (K_\perp \nabla_\perp T + K_\parallel \nabla_\parallel T) + S_T
\]

\[
e_\phi \cdot \nabla \times \left\{ \rho \frac{\partial v}{\partial t} = -\rho (v \cdot \nabla) v - \nabla p + j \times B + \mu \Delta v \right\}
\]

\[
B \cdot \left\{ \rho \frac{\partial v}{\partial t} = -\rho (v \cdot \nabla) v - \nabla p + j \times B + \mu \Delta v \right\}
\]
JOREK: reduced MHD

- JOREK solves nonlinear reduced MHD equations in toroidal geometry

\[
\begin{align*}
\frac{\partial \Psi}{\partial t} &= \eta j - R \left[ u, \Psi \right] - F_0 \frac{\partial u}{\partial \phi} \\
\frac{\partial \rho}{\partial t} &= -\nabla \cdot (\rho v) + \nabla \cdot (D_{\perp} \nabla_{\perp} \rho) + S_\rho \\
\frac{\partial (\rho T)}{\partial t} &= -v \cdot \nabla (\rho T) - \gamma \rho T \nabla \cdot v + \nabla \cdot (K_{\perp} \nabla_{\perp} T + K_{||} \nabla_{||} T) + S_T \\
e_{\phi} \cdot \nabla \times \left\{ \rho \frac{\partial v}{\partial t} = -\rho (v \cdot \nabla) v - \nabla p + j \times B + \mu \Delta v \right\} \\
B \cdot \left\{ \rho \frac{\partial v}{\partial t} = -\rho (v \cdot \nabla) v - \nabla p + j \times B + \mu \Delta v \right\} \\
\quad \quad \quad \quad \quad \quad \quad j \equiv -j_{\phi} = \Delta^* \Psi \\
\omega \equiv -\omega_{\phi} = \nabla_{\text{pol}}^2 u
\end{align*}
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\[
\omega \equiv -\omega_\phi = \nabla_{\text{pol}}^2 u
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Definitions: \( \mathbf{B} \equiv \frac{F_0}{R} \mathbf{e}_\phi + \frac{1}{R} \nabla \Psi \times \mathbf{e}_\phi \) and \( v \equiv -R \nabla u \times \mathbf{e}_\phi + v_\parallel \mathbf{B} \)
JOREK: reduced MHD

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\]

\[
\vec{e}_\phi \cdot \nabla \times \left\{ \rho \frac{\partial \vec{v}}{\partial t} = -\rho (\vec{v} \cdot \nabla) \vec{v} - \nabla p + j \times \vec{B} + \mu \Delta \vec{v} \right\}
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\]

\[
j \equiv -j_\phi = \Delta^* \Psi
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\[
\omega \equiv -\omega_\phi = \nabla^2_{\text{pol}} u
\]

Definitions: \( \vec{B} \equiv \frac{F_0}{R} \vec{e}_\phi + \frac{1}{R} \nabla \Psi \times \vec{e}_\phi \) and \( \vec{v} \equiv -R \nabla u \times \vec{e}_\phi + \nu_\parallel \vec{B} \)

Variables: \( \Psi, u, \nu_\parallel, \rho, T, j, \omega \)
**Discretization**

- **poloidal plane**: 2D Bézier finite elements

\[
P(s, t) = \sum_{i=0}^{3} \sum_{j=0}^{3} P_{ij} B_i(s) B_j(t)
\]

- **toroidal direction**: Fourier decomposition
- **fully implicit time stepping**
Grid generation

- equilibrium is computed on initial polar grid
- flux surface aligned X-point grid is generated
- grid can be refined in the regions of interest
Introduction

Grid generation

▷ equilibrium is computed on initial polar grid
▷ flux surface aligned X-point grid is generated
▷ grid can be refined in the regions of interest

Boundary conditions

▷ ideally conducting wall and modified Bohm
Introduction

Edge-localized modes

- Relaxation-oscillation instability at edge of H-mode plasmas
- Driven by large edge pressure gradient & edge current density
- Eject energy & particles from plasma
- Relevant for future fusion devices
  - Help to control particle & impurity content
  - High heat fluxes can damage plasma facing components

→ Theoretical comprehension of ELMs is crucial to predict and control ELM properties
Introduction

Experimental observations

- **linear theory**: intermediate toroidal mode numbers are most unstable
Introduction

- **linear theory**: intermediate toroidal mode numbers are most unstable
- **recent experimental observations (TCV)**: toroidal mode structure often dominated by low-n components

![Fourier spectrum of measured ELM](image)

Example of measured ELM Fourier spectrum

Dominant toroidal components in ELMy discharge

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ELM simulations

Parameters & geometry

- simulations are based on typical type-I ELMy ASDEX Upgrade discharge
  - plasma parameters based on ASDEX Upgrade, but larger resistivity ($S \approx 10^5$)
  - ASDEX Upgrade geometry including separatrix, X-point and open field lines
ELM simulations

- simulations are based on typical type-I ELMy ASDEX Upgrade discharge
  - plasma parameters based on ASDEX Upgrade, but larger resistivity ($S \approx 10^5$)
  - ASDEX Upgrade geometry including separatrix, X-point and open field lines
- large set of included toroidal Fourier harmonics ($n = 1, 2, \ldots, 16$)
ELM simulations

Evolution of the perturbation

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ELM simulations

Evolution of the perturbation

linear phase
ELM simulations

Evolution of the perturbation

linear phase $\rightarrow$ early nonlinear phase
ELM simulations

Evolution of the perturbation

linear phase  $\rightarrow$ early nonlinear phase  $\rightarrow$ saturation

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Interpretation

Simple quadratic coupling model

Idea: ”sum & difference mode number generation”
Simple quadratic coupling model

Interpretation

Idea: "sum & difference mode number generation"

- superposition of harmonics $j$ & $k$ $\rightarrow$ generation of $i = |j \pm k|$
Interpretation

Simple quadratic coupling model

Idea: "sum & difference mode number generation"

- superposition of harmonics $j$ & $k$ $\xrightarrow{\text{quadratic terms}}$ generation of $i = \lvert j \pm k \rvert$

$\Rightarrow$ time evolution of amplitude $A_i$

$$\frac{\partial A_i}{\partial t} = \gamma_i A_i + \gamma_{jk} A_j A_k$$

linear growth    coupling
Interpretation

Simple quadratic coupling model

Idea: "sum & difference mode number generation"

\[ \Delta \text{superposition of harmonics } j \& k \xrightarrow{\text{quadratic terms}} \text{generation of } i = |j \pm k| \]

\[ \Rightarrow \text{time evolution of amplitude } A_i \]

\[ \frac{\partial A_i}{\partial t} = \gamma_i A_i + \gamma_{jk}^i A_j A_k \]

\( \gamma_i \): linear growth rate

\[ \leftrightarrow \text{constant } \Rightarrow \text{no saturation effects included} \]
Interpretation

Simple quadratic coupling model

Idea: ”sum & difference mode number generation”

- superposition of harmonics $j$ & $k$ $\rightarrow$ generation of $i = |j \pm k|$

$\implies$ time evolution of amplitude $A_i$

$$\frac{\partial A_i}{\partial t} = \gamma_i A_i + \gamma^i_{jk} A_j A_k$$

$\gamma_i$: linear growth rate

$\Rightarrow$ constant $\Rightarrow$ no saturation effects included

$\gamma^i_{jk}$: coupling constant

$\Rightarrow$ constant $\Rightarrow$ mode rigidity assumed
Simple quadratic coupling model

\[ \frac{\partial A_i}{\partial t} = \gamma_i A_i + \sum_{j=1}^{16} \sum_{k=1}^{16} \gamma_{jk}^i A_j A_k \delta(i \pm j \pm k) \]

\[ \text{for a set of harmonics } i = 1, 2, ..., 16 \]

\[ \text{set of coupled nonlinear differential equations reproduces evolution of toroidal Fourier spectrum in JOREK simulations} \]
Interpretation

Simple quadratic coupling model

\[ \frac{\partial A_i}{\partial t} = \gamma_i A_i + \sum_{j=1}^{16} \sum_{k=1}^{16} \gamma_{jk} A_j A_k \delta(i \pm j \pm k) \]

\( \downarrow \) for a set of harmonics \( i = 1, 2, \ldots, 16 \)

\( \uparrow \) set of coupled nonlinear differential equations reproduces evolution of toroidal Fourier spectrum in JOREK simulations

\( \uparrow \) relevant coupling constants: \( \gamma_{9,10}^1, \gamma_{8,10}^2, \gamma_{7,10}^3, \gamma_{6,10}^4, \gamma_{7,8}^{15}, \gamma_{7,9}^{16} \)
Interpretation

Results of simple model

The simple quadratic coupling model reproduces JOREK results in the early nonlinear phase. This model provides an explanation for the strong low-n components observed in experiments.

![Graph showing energy vs. time step for different toroidal mode numbers.](image-url)
simple quadratic coupling model reproduces JOREK results in early nonlinear phase

model gives explanation for strong low-n components in experiments
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Linearly unstable $n = 1$ extends over a large part of the plasma core.
Interpretation

Localization of driven harmonics

- linearly unstable \( n = 1 \) extends over a large part of the plasma core
- nonlinearly driven \( n = 1 \) is localized at plasma edge (where driving harmonics are maximal and in phase)
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Summary...

- nonlinear reduced MHD **ELM simulations** based on ASDEX Upgrade
- large set of included toroidal harmonics
- **subdominant low-n harmonics** become important due to **nonlinear drive**
- \( n = 1 \) reaches energies comparable to linearly dominant harmonics
- **correspondence to experimental observations** of dominant low-n components
- **simple quadratic interaction model** reproduces and explains early nonlinear evolution of toroidal harmonics in JOREK simulations
- spatial structure of \( n = 1 \) becomes localized at edge when nonlinearly driven

… and Outlook

- enable more realistic resistivity
- analyze how nonlinear interaction of toroidal harmonics is influenced by
  - diamagnetic drift effects
  - sheared toroidal plasma rotation
Thank you for your attention!

References

Simulations

Experiment

JOREK

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Max-Planck/Princeton Center for Plasma Physics
HELIOS at IFERC-CSC
Simple coupling model  

Energy conservation

\[ \frac{\partial A_i}{\partial t} = \gamma_i A_i + \sum_{j=1}^{16} \sum_{k=1}^{16} \gamma_{jk}^i A_j A_k \delta(i \pm j \pm k) \quad \text{for } i = 1, 2, ..., 16 \]

- linear terms \( \rightarrow \) influx of energy
- nonlinear terms \( \rightarrow \) exchange of energy between different harmonics (total energy should be conserved)

\[ 0 = \frac{\partial E_{tot}}{\partial t} \propto \frac{\partial}{\partial t} \sum_i A_i^2 \quad (\text{only nonlinear terms}) \]

\( \Rightarrow \) additional constraints for the coupling constants (12 free parameters remain)
Simple coupling model

Energy conservation

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