Modelling of wall currents excited by plasma wall-touching kink and vertical modes during a tokamak disruption, with application to ITER

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17\(^{th}\) European Fusion Theory Conference, Athens - Greece

October 9-12, 2017
Overview

1. Introduction & assumptions
2. Two kinds of surface currents in the thin wall
3. Energy principle for the thin wall currents
4. Matrix circuit equations for triangle wall representation
5. Simulations of Source/Sink Currents (SSC)
   - Numerical solution
   - Analytical solution
6. Next steps
7. Summary
8. References
1. Introduction & assumptions

- the nonlinear evolution of MHD instabilities - the Wall Touching Kink Modes (WTKM) - leads to a dramatic quench of the plasma current within $ms \rightarrow$ very energetic electrons are created (runaway electrons) and finally a global loss of confinement happens $\equiv$ a major disruption;

- in the ITER tokamak, the occurrence of a limited number of major disruptions will definitively damage the chamber with no possibility to restore the device;

- the WTKM are frequently excited during the Vertical Displacement Event (VDE) and cause big sideways forces on the vacuum vessel [1, 2].

- objective: to consider in JOREK, STARWALL, JOREK-STARWALL the current exchange plasma-wall-plasma
Theoretical example: modelling of an axisymmetric vertical instability [Zakharov et. al, PoP (2012)].

Theoretically simplest example of vertically unstable plasma:

1. Quadrupole field of external PFCoils
2. Straight plasma column with uniform current along z-axis
3. Elliptical cross-section
4. Plasma is shifted downward from equilibrium
5. Plasma current is attracted by the nearest PFCoil with the same current direction \(\equiv\) instability

**Question**: Where the plasma will go to?

The answer isn’t trivial!
| Initial downward plasma displacement | Nonlinear phase of instability. Negative surface current at the leading plasma side | 1. Strong negative sheet current at the leading plasma edge  
2. Plasma cross-section becomes triangle-like |

(a) opposite poloidal field $B_\theta^{\text{vac}} \approx -B_\theta^{\text{core}}$ across the leading plasma edge;  
(b) two Null Y-points of poloidal field in the triangle-like plasma cross-section. Plasma should be leaked through the Y-point until full disappearance.  

**Strong external field stops vertical motion.**
1) **Free boundary MHD modes**, which are always associated with the surface currents, are evident in the tokamak disruptions:
(a) excitation of m/n=1/1 kink mode during VDE on JET (1996),
(b) recent measurements of Hiro currents on EAST (2012).
2) Both theory and JET, EAST experimental measurements indicate that the galvanic contact of the plasma with the wall is critical in disruption;
3) **The thin wall approximation** is reasonable for thin stainless steel structures of the vacuum vessel ( # 1-3 cm & $\sigma = 1.38 \cdot 10^{-6} \Omega^{-1}m^{-1}$.)
4) For simulating the plasma-wall interaction during disruption, the reproduction of 3D structure of the wall is important (e.g., the galvanic contact is sensitive to the local geometry of the wall in the wetting zone [3].
5) Our **wall model** covers both eddy currents, excited inductively, and source/sink currents due to current sharing between plasma and wall.
6) We adopted a **FE triangle representation** of the plasma facing wall surface (- *simplicity* & - *analytical formulas for B* of a uniform current in a single triangle) [4].
2. Two kinds of surface currents in the thin wall

- **Helmholtz decomposition theorem** states that any sufficiently smooth, rapidly decaying vector field \( \mathbf{F} \), twice continuously differentiable in 3D, can be resolved into the sum of an irrotational (curl-free) vector field and a solenoidal (divergence-free) vector field;

- thus, the **surface current density** \( h_j \) in the conducting shell can be split into two components: [3]

\[
\begin{align*}
  h_j &= i - \bar{\sigma} \nabla \phi^S, \\
  i &\equiv \nabla I \times \mathbf{n}, \quad (\nabla \cdot i) = 0, \quad \bar{\sigma} \equiv h\sigma, \\
\end{align*}
\]

(a) \( i \) = the divergence free surface current (eddy currents) and
(b) \( -\bar{\sigma} \nabla \phi^S \) = the source/sink current (S/SC) with potentially finite \( \nabla \cdot \) in order to describe the current sharing between plasma and wall,

\( \bar{\sigma} = h\sigma \) = surface wall conductivity, \( h \) = thickness of the current distrib.,

\( I \) = the stream function of the divergence free component (eddy currents),

\( \mathbf{n} \) = unit normal vector to the wall,

\( \phi^S \) = the source/sink potential (\( \equiv \) surface function).
• The S/S-current in Eq. (1) is determined from the **continuity equation** of the S/S currents across the wall

\[
\nabla \cdot (h \mathbf{j}) = - \nabla \cdot (\bar{\sigma} \nabla \phi^S) = j_\perp, \quad (2)
\]

• \( j_\perp \equiv -(\mathbf{j} \cdot \mathbf{n}) \) = the density of the current coming from/to the plasma, \( j_\perp > 0 \) for \( j_\perp \) plasma \( \rightarrow \) wall.
• Faraday law gives

\[
- \frac{\partial \mathbf{A}}{\partial t} - \nabla \phi^E = \bar{\eta} (\nabla \mathbf{l} \times \mathbf{n}) - \nabla \phi^S, \quad \bar{\eta} \equiv \frac{1}{\bar{\sigma}} \quad (3)
\]

\( \mathbf{A} = \text{vec. pot. of} \; \mathbf{B}, \; \phi^E = \text{electric potential}, \; \bar{\eta} = \text{effective resistivity}. \)
• Eqs. (2, 3) describe the current distribution in the thin wall, given the sources \( j_\perp, B_{pl}\perp, B_{coil}\perp \) as \( f(\mathbf{x}, t) \);
• Eq. (2) for \( \phi^S \) is independent from Eq. (3), but contributes via \( \partial B_{\perp}^S / \partial t \) to the r.h.s. of Eq. (3).
• for our numerical wall model, $A$ can be calculated with:

$$A^{\text{wall}}(r) = A^{\text{l}}(r) + A^{\text{S}}(r) = \sum_{i=0}^{N_T-1} (h_j)_i \int \frac{dS_i}{|r - r_i|},$$

(4)

with $\sum$ over the $N_T$ FE triangles and the $\int$ is taken over $\Delta$ surface analytically.

• the equation for the stream function $I$ is given by [4, 5]

\[
\nabla \cdot (\frac{1}{\sigma} \nabla I) = \frac{\partial B_\perp}{\partial t} = \frac{\partial (B_{\perp}^{\text{pl}} + B_{\perp}^{\text{coil}} + B_{\perp}^{\text{l}} + B_{\perp}^{\text{S}})}{\partial t}
\]

(5)

$B_{\perp}^{\text{pl,coil,l,S}}$ = the perpendicular to the wall $B$ component.

• Biot-Savart relation for $B$ is necessary to close the system of Eqs..
3. Energy principle for the thin wall currents

- $\phi^S$ can be obtained by minimizing the functional $W^S$ [3].

$$W^S = \int \left\{ \frac{\bar{\sigma}(\nabla \phi^S)^2}{2} - j_\perp \phi^S \right\} dS - \oint \phi^S \bar{\sigma}[(n \times \nabla \phi^S) \cdot d\vec{\ell}] \tag{6}$$

- $\int dS$ is taken along the wall surface,
- $\oint d\vec{\ell}$ is taken along the edges of the conducting surfaces with the integrand representing the surface current normal to the edges,
- $\oint d\vec{\ell}$ takes into account the external voltage applied to the edges of the wall and $= 0$, as happens in typical cases.
- $I$ can be obtained by minimizing the functional $W'$ [3]

\[
W' \equiv \frac{1}{2} \int \left\{ \frac{\partial (i \cdot A')}{\partial t} + \frac{1}{\sigma} |\nabla I|^2 \right. \\
+ 2 \left( i \cdot \frac{\partial A^{\text{ext}}}{\partial t} \right) \right\} dS - \oint (\phi^E - \phi^S) \frac{\partial I}{\partial \ell} d\ell. \tag{7}
\]
4. Matrix circuit equations for triangle wall representation

- the two energy functionals for $\phi^S$ and for $I$ are suitable for implementation into numerical codes and constitute the electromagnetic wall model for the wall touching kink and vertical modes;

- the substitution of $I, \phi^S$ as a set of plane functions inside triangles leads to the finite element representation of $W^I, W^S$ as quadratic forms for unknowns $I, \phi^S$ in each vertex;

- the unknowns vectors at the $N_V$ vertexes are

\[
\vec{I} \equiv I_0, I_1, \ldots, I_{N_V-1}, \quad (8) \\
\vec{\phi^S} \equiv \phi^S_0, \phi^S_1, \ldots, \phi^S_{N_V-1}.
\]
\begin{itemize}
  \item the minimization of quadratic forms $W^S$ and $W^I$
  \[ \partial W^S / \partial \phi^S = 0, \quad \partial W^I / \partial \bar{I}^n = 0, \quad \partial W^I / \partial \phi^S = 0, \]
  \end{itemize}

leads to linear systems of equations with Hermitian symmetric-positive definite matrices which can be solved using the Cholesky decomposition: $W = L \cdot L^*$

- lower triangular
- conjugate transpose of $L$

\begin{itemize}
  \item the matrix equations are [6]
  \end{itemize}

\[ W^{SS} \cdot \phi^S = -\vec{j}_\perp \]
\[ M^{II} \cdot \frac{\bar{I}^n - \bar{I}^{n-1}}{\Delta t} + R \cdot (\bar{I}^n - \bar{I}^{n-1}) + R \cdot \bar{I}^{n-1} + W^{IS} \cdot \frac{\phi^S,n - \phi^S,n-1}{\Delta t} \]

\[ = -A^{IV} \cdot \frac{\partial (\vec{A}^{pl} + \vec{A}^{ext})}{\partial t}, \]

(9)

with vector sources $\vec{j}_\perp \equiv \{j_\perp,0, j_\perp,1, j_\perp,2, \ldots j_\perp,N_{V-1}\}$ and $\vec{A}^{pl,ext} \equiv \{\vec{A}_0^{pl,ext}, \vec{A}_1^{pl,ext}, \vec{A}_2^{pl,ext}, \ldots \vec{A}_{N_{V-1}}^{pl,ext}\}$, with $\Delta t =$ the “wall-time-step”, superscript $n =$ time slice.
• inverting the matrices $W^{SS}$ and $M^{II}$ the calculation of the wall currents is reduced to 2 relations implemented in our code

\[ \vec{\phi}^S = - \left( W^{SS} \right)^{-1} \cdot \vec{j}_\perp \]
\[ \vec{l}^n = \vec{l}^{n-1} \cdot \hat{R} \cdot \vec{l}^{n-1} \Delta t + \hat{W}^{IS} \cdot \frac{\partial \vec{j}_\perp}{\partial t} \Delta t \]
\[ - A^{IV} \cdot \frac{\partial (\vec{A}^{pl} + \vec{A}^{ext})}{\partial t} \Delta t. \] (10)

• as output, the code returns the values of $\phi_i^S$ and $l_i$ in all vertexes, allowing the calculation of the $A$ and $B$ of the wall currents in any point $\vec{r}$.
5. Simulation of Source/Sink Currents (SSC)

5.1. Numerical solution [6, 7]

Fig.2 Identifying the FE edge elements in ITER wall  Fig.3 21744 FE triangle distribution in ITER wall
<table>
<thead>
<tr>
<th>iVertex</th>
<th>$\sigma*1e-6$</th>
<th>$h$ [m]</th>
<th>$x$ [m]</th>
<th>$y$ [m]</th>
<th>$z$ [m]</th>
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<td>0.0300</td>
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**Table 1.** Vertexes, thickness $h$ and $\sigma$ distributions for the ITER wall.
Table 2. *Triangles and correspondent vertexes distribution for the ITER wall.*

<table>
<thead>
<tr>
<th>iTriangle</th>
<th>i[A]</th>
<th>i[B]</th>
<th>i[C]</th>
<th>Prop</th>
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<tr>
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</tr>
<tr>
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</tr>
<tr>
<td>---------------------</td>
<td>-----------------</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$(\mathbf{W}^{SS})^{-1}$</td>
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<td></td>
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<tr>
<td>$\hat{\mathbf{R}}$</td>
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<tr>
<td>$\hat{\mathbf{A}}^{IV}$</td>
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<tr>
<td>$(\hat{\mathbf{M}}^{II}_{\sigma})^{-1}$</td>
<td>855,106</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 3.** Matrices size for the 21744 triangles and 11223 vertexes of the FE discretization of ITER wall.
Fig. 4 Wetting zone created by a VDE and a kink \( m/n=1/1 \). The color of the wall =distribution of the perturbed \( A\varphi \).

Fig. 5 Eddy currents excited by the plasma perturbation. The color corresponds to \( I \) stream function.

Fig. 6 Eddy currents excited by both plasma perturbation and S/S current. Distribution of \( \Phi^S \) at the wall surface.

Fig. 7 Total surface current with the S/S current as the dominant component.
6.2. Analytical solution

- For a shell with elliptical cross-section and three holes (Fig. 8.1 with the correspondent geometry in a curvilinear coordinate system \((u, v)\) in Fig. 8.2). For \(h\sigma = 1\), we have to solve the eq.

\[
\nabla^2 \phi^S = j_\perp(u, v) \quad u = \text{toroidal coord.}, \quad v = \text{poloidal coord.},
\]

with pure homogeneous Neumann B.C. and the following existence condition to be satisfied:

\[
\int_\Omega j_\perp \, d\Omega = \int_{\partial \Omega} \nabla \phi^S \cdot n \, dS
\]

\[
\Omega = \Omega_e \setminus \Omega_i \quad \partial \Omega = \Gamma_e \cup \Gamma_i
\]

The analytical \(\phi(u, v)\) has been chosen in the form [3, 5, 7]

\[
\phi^S(u, v) = \int G_u(u) \, du \cdot \int G_v(v) \, dv, \quad \text{with}
\]

\[
G_u(u) = \prod(u - u_{ik}); \quad G_v(v) = \prod(v - v_{ik}); \quad i = 0, \ldots, 3, \quad k = 1, 2,
\]

If for 1 hole the relative error was of 0.003 for a grid with a mesh \(32 \times 32 \times 4\), for 3 holes the error is \(\approx 5\) times greater.
Fig. 8.1 Tokamak wall with elliptical cross-section and three holes (in blue).

Fig. 8.2 Multiply connected test domain $D(u,v)$ between the four rectangles in a curvilinear coordinate system $(u,v)$.

Fig. 8.3 Distribution of the analytical $\Phi_S(u,v)$ function.
6. Next steps

- to realize the connection with JOREK in order to obtain the following input data:

\[ \vec{A}^\text{pl} + \vec{A}^\text{ext} = f(t, r) \]
\[ \vec{J}_\perp = f(t, r) \]
\[ \Delta t \]


- to include non-symmetrical wall structures

7. Summary

- a rigorous formulation of the surface current eqs. was formulated;

- in the triangular representation of the wall surface, both surface current components are represented by the same model of a uniform current density inside each $\Delta$;

- the coupling of finite element matrix equations for both types of currents contains the same matrix elements of mutual capacitance $C_{ij}$ of two triangles $\Delta_{i,j}$ which can be calculated analytically;

- our model has been checked successfully on an analytical case;

- our code received the status of ”open source license”.
8. References